

## Algoritmi numerici pentru analiza circuitelor electrice rezistive neliniare

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Departamentul de Electrotehnică

Suport didactic pentru disciplina *Metode numerice*,  
Facultatea de Inginerie Electrică, 2017-2018

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## Cuprins

- 1 Introducere
  - Elemente de circuit rezistive neliniare
  - Formularea problemei
  - Ecuații
  - Exemple
- 2 Metoda nodală clasică
- 3 Descrierea caracteristicilor neliniare
  - Prin cod
  - Prin date
- 4 Algoritmi
  - Metoda Newton
  - Idei de implementare
  - Preprocesare
  - Procesare

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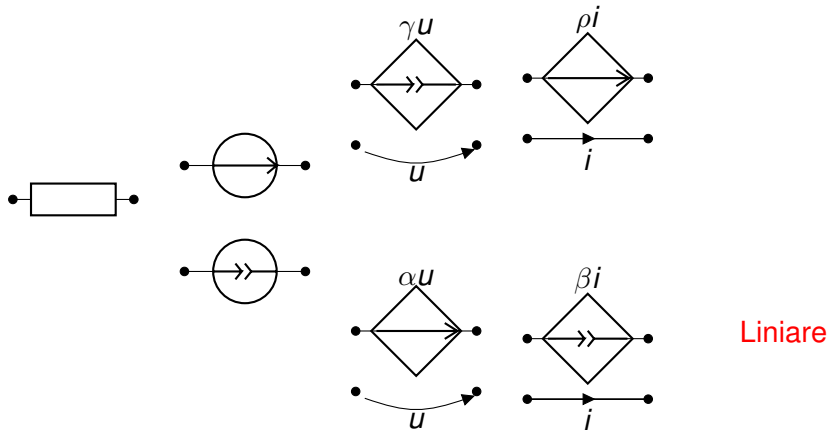
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## Elemente ideale - rezistive, liniare



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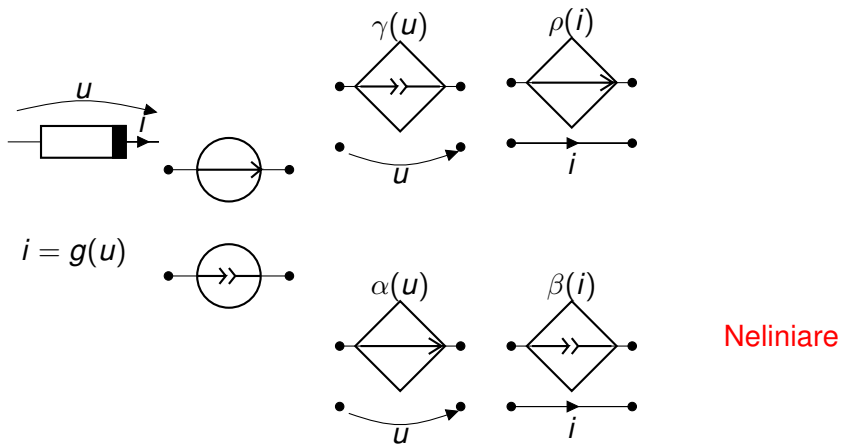
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## Elemente ideale - rezistive, neliniare



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## Elemente reale - rezistive, neliniare

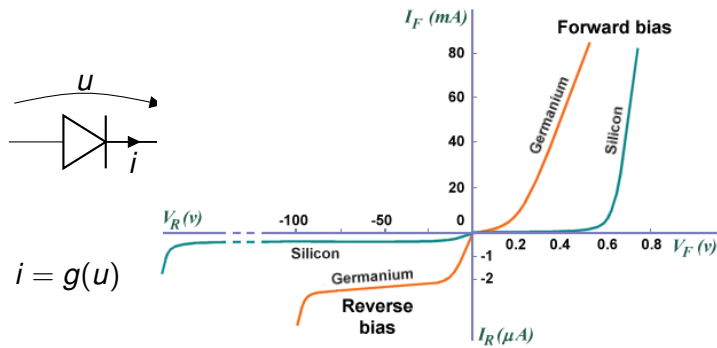


Figura este preluată de la  
<https://www.technologyuk.net/physics/>

## Elemente reale - rezistive, neliniare

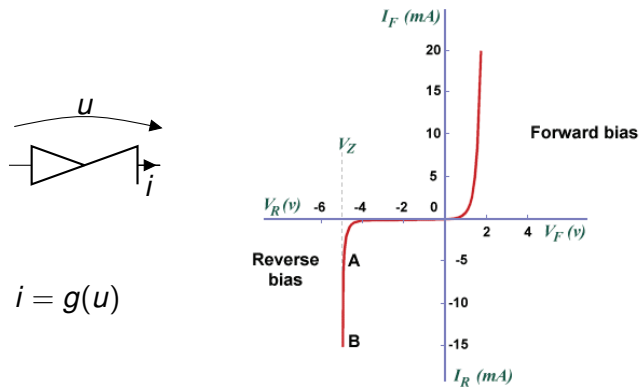


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## Analiza circuitelor electrice rezistive neliniare (c.c.)

### Date:

- *Topologia circuitului* (graful circuitului) - poate fi descris:
  - geometric;
  - numeric (matrice topologice/ *netlist*);
- Pentru fiecare latură liniară  $k$ :
  - tipul laturii (R,SUCU,SICI,SICU,SUCI, SIT,SIC);
  - caracteristica constitutivă
    - $R_k$ ;
    - parametrul de transfer  $\alpha_k, \beta_k, \gamma_k, \rho_k$ ;
    - semnalul de comandă (curent/tensiune, latură/noduri);
    - parametrii surselor:  $(e_k, j_k)$

## Analiza circuitelor electrice rezistive neliniare (c.c.)

- Pentru fiecare latură neliniară  $k$ :
  - tipul laturii (Rn,SUCUn,SICIn,SICUn,SUCIn);
  - caracteristica constitutivă neliniaa
    - $f_k(i)$  dacă controlul este în curent sau  $g_k(u)$  dacă controlul este în tensiune;
    - dependențele  $\alpha_k(u), \beta_k(i), \gamma_k(u), \rho_k(i)$ ;
    - semnalul de comandă (curent/tensiune, latură/noduri);

**Se cer:**  $i_k(t), u_k(t), k = 1, 2, \dots, L$ .

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## Ca la c.c. - cazul elementelor liniare

- 1 Kirchhoff I
- 2 Kirchhoff II
- 3 Ecuatii constitutive pentru elementele rezistive liniare:
  - laturi de tip SRC, SRT;
  - laturi de tip SIC, SIT;
  - laturi de tip SUCU, SICI, SUCI, SICU - comandate liniar.

relatii algebrice

**DAR**

## Elementele rezistive neliniare

Ecuatii constitutive pentru elementele rezistive neliniare:

- rezistoare neliniare;
- surse comandate neliniar;

relatii algebrice neliniare

Sistemul de rezolvat va fi un sistem algebric neliniar

Ce se întâmplă dacă surselor independente sunt variabile în timp?

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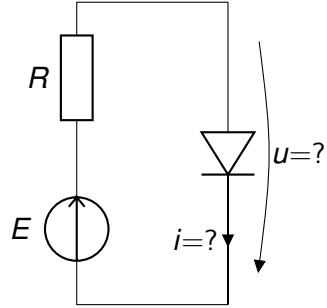
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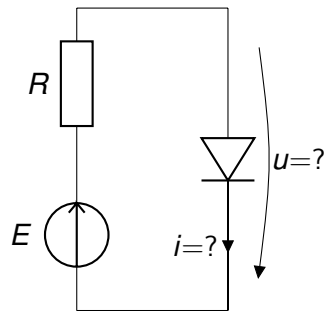
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## Exemplul 1



## Exemplul 1



$$i = g(u)$$
$$i = \frac{E - u}{R}$$

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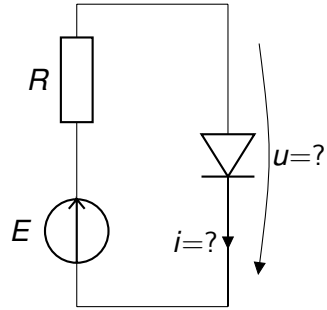
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## Exemplul 1



$$i = g(u)$$
$$i = \frac{E - u}{R}$$

$E = 1.25\text{V}, R = 1.25\text{m}\Omega$

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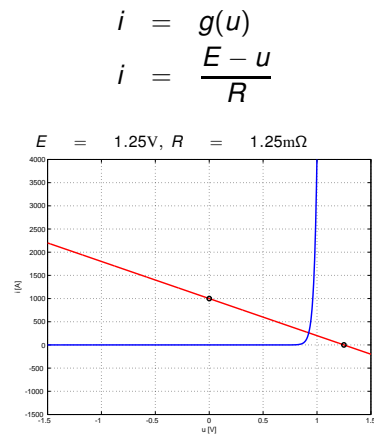
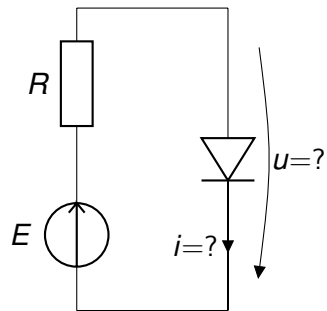
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## Exemplul 1



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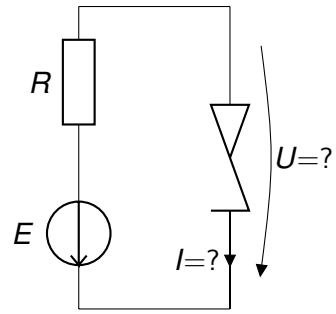
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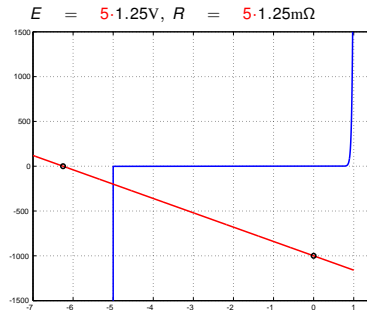


### Exemplul 3 b)



$$i = g(u)$$

$$i = \frac{-E - u}{R}$$



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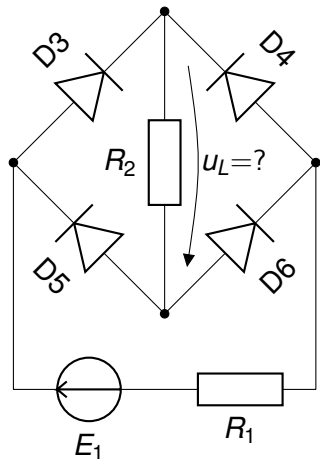
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### Exemplul 4



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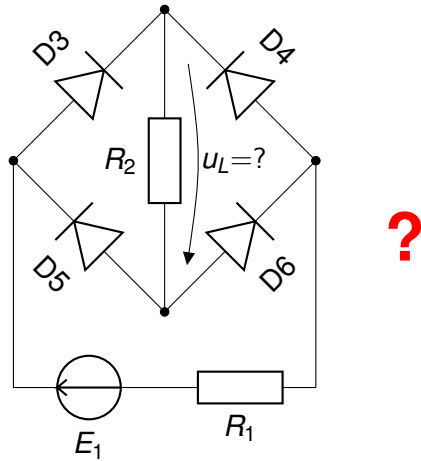
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## Exemplul 4



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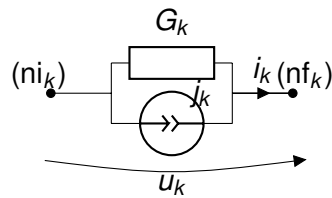
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## Laturi controlate în tensiune

Cazul liniar (SRC)



$$i_k = G_k u_k + j_k$$

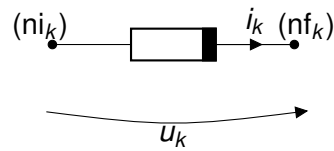
$$\mathbf{i} = \mathbf{G}\mathbf{u} + \mathbf{j}$$

$$\mathbf{G} = \text{diag}\{G_1, G_2, \dots, G_L\}$$

$$\mathbf{G} \in \mathbb{R}^{L \times L}$$

$$\mathbf{u}, \mathbf{j}, \mathbf{i} \in \mathbb{R}^{L \times 1}$$

Cazul neliniar



$$i_k = g_k(u_k)$$

$$\mathbf{i} = \mathbf{G}(\mathbf{u})$$

$$\mathbf{G} = [g_1, g_2, \dots, g_L]^T$$

$$\mathbf{G} : \mathbb{R}^L \rightarrow \mathbb{R}^L$$

$$\mathbf{u}, \mathbf{i} \in \mathbb{R}^{L \times 1}$$

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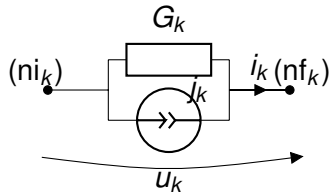
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## Laturi controlate în tensiune

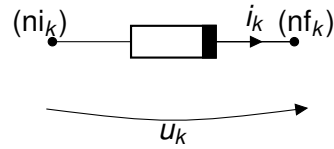
Cazul liniar (SRC)



$$i_k = G_k u_k + j_k$$

$$\begin{aligned} \mathbf{i} &= \mathbf{G}\mathbf{u} + \mathbf{j} \\ \mathbf{A}\mathbf{i} &= \mathbf{0} \\ \mathbf{u} &= \mathbf{A}^T \mathbf{V} \\ \mathbf{A}(\mathbf{G}\mathbf{A}^T \mathbf{V} + \mathbf{j}) &= \mathbf{0} \end{aligned}$$

Cazul neliniar



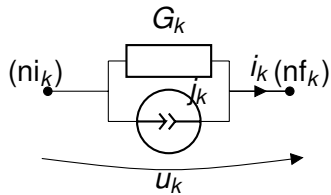
$$i_k = g_k(u_k)$$

$$\begin{aligned} \mathbf{i} &= \mathbf{G}(\mathbf{u}) \\ \mathbf{A}\mathbf{i} &= \mathbf{0} \\ \mathbf{u} &= \mathbf{A}^T \mathbf{V} \\ \mathbf{A}(\mathbf{G}(\mathbf{A}^T \mathbf{V})) &= \mathbf{0} \end{aligned}$$

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## Laturi controlate în tensiune

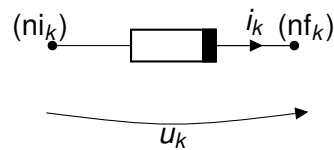
Cazul liniar (SRC)



$$i_k = G_k u_k + j_k$$

$$\begin{aligned} \mathbf{i} &= \mathbf{G}\mathbf{u} + \mathbf{j} \\ \mathbf{A}\mathbf{i} &= \mathbf{0} \\ \mathbf{u} &= \mathbf{A}^T \mathbf{V} \\ \mathbf{A}\mathbf{G}\mathbf{A}^T \mathbf{V} &= -\mathbf{A}\mathbf{j} \end{aligned}$$

Cazul neliniar



$$i_k = g_k(u_k)$$

$$\begin{aligned} \mathbf{i} &= \mathbf{G}(\mathbf{u}) \\ \mathbf{A}\mathbf{i} &= \mathbf{0} \\ \mathbf{u} &= \mathbf{A}^T \mathbf{V} \\ \mathbf{A}\mathbf{G}(\mathbf{A}^T \mathbf{V}) &= \mathbf{0} \end{aligned}$$

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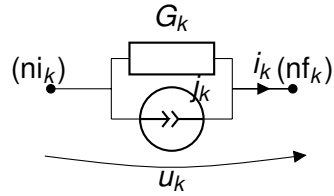
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## Laturi controlate în tensiune

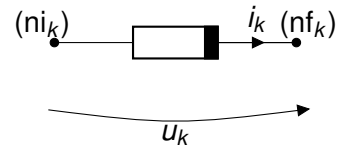
Cazul liniar (SRC)



$$i_k = G_k u_k + j_k$$

**$AGA^T \mathbf{V} = -\mathbf{A}j$**   
 Sistem algebraic liniar

Cazul neliniar



$$i_k = g_k(u_k)$$

**$AG(A^T \mathbf{V}) = \mathbf{0}$**   
 Sistem algebraic neliniar  
 **$\mathbf{F}(\mathbf{V}) = \mathbf{0}$**  unde  
 **$\mathbf{F}(\mathbf{V}) = AG(A^T \mathbf{V})$**   
 **$\mathbf{F} : \mathbb{R}^{(N-1)} \rightarrow \mathbb{R}^{(N-1)}$**

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## Dioda semiconductoră

Modelul exponențial (de exemplu modelul cu parametrii  $I_s$  și  $u_T$ )

$$i(u) = I_s \left( e^{\frac{u}{u_T}} - 1 \right)$$

unde  $I_s \approx 10^{-6} \text{A}$ ,  $u_T \approx 25 \text{mV}$

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## Dioda semiconductoare

Modele liniare pe porțiuni (de exemplu - modelul cu parametrii  $u_p$ ,  $G_d$ ,  $G_i$ ) definite prin cod

$$i(u) = \begin{cases} G_i u & \text{dacă } u \leq u_p \\ G_d(u - u_p) + G_i u_p & \text{dacă } u > u_p \end{cases}$$

## Dioda semiconductoare

Modele liniare pe porțiuni - definite prin tabele de valori

Exemplu - modelul lpp cu parametrii  $u_p$ ,  $G_d$ ,  $G_i$

$u$	0	$u_p$	$2u_p$
$i$	0	$G_i u_p$	$(G_i + G_d)u_p$

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## Newton

Iterații Newton:

- **Ecuatie:**  $f(x) = 0$

$$x^{(m+1)} = x^{(m)} - f(x^{(m)})/f'(x^{(m)})$$

sau

$$z = f(x^{(m)})/f'(x^{(m)}) \quad (1)$$

$$x^{(m+1)} = x^{(m)} + z \quad (2)$$

- **Sistem:**  $\mathbf{F}(\mathbf{x}) = \mathbf{0}$

$$\mathbf{x}^{(m+1)} = \mathbf{x}^{(m)} - (\mathbf{F}'(\mathbf{x}^{(m)}))^{-1} \mathbf{F}(\mathbf{x}^{(m)})$$

sau

$$\mathbf{F}'(\mathbf{x}^{(m)})\mathbf{z} = \mathbf{F}(\mathbf{x}^{(m)}) \quad (3)$$

$$\mathbf{x}^{(m+1)} = \mathbf{x}^{(m)} + \mathbf{z} \quad (4)$$

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## Newton

În cazul circuitelor rezistive neliniare  $\mathbf{F}(\mathbf{V}) = \mathbf{0}$  unde

$$\mathbf{F}(\mathbf{V}) = \mathbf{AG}(\mathbf{A}^T \mathbf{V})$$

Iterații Newton:

$$\mathbf{F}'(\mathbf{V}^{(m)})\mathbf{z} = -\mathbf{F}(\mathbf{V}^{(m)}) \quad (5)$$

$$\mathbf{V}^{(m+1)} = \mathbf{V}^{(m)} + \mathbf{z} \quad (6)$$

$$\mathbf{F}'(\mathbf{V}) = \mathbf{AG}'(\mathbf{A}^T \mathbf{V})\mathbf{A}^T$$

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## Newton

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- Calculul Jacobianului necesită evaluarea conductanțelor dinamice!

## Newton

În cazul circuitelor rezistive neliniare  $\mathbf{F}(\mathbf{V}) = \mathbf{0}$  unde

$$\mathbf{F}(\mathbf{V}) = \mathbf{AG}(\mathbf{A}^T \mathbf{V})$$

Iterații Newton:

$$\mathbf{F}'(\mathbf{V}^{(m)})\mathbf{z} = -\mathbf{F}(\mathbf{V}^{(m)}) \quad (5)$$

$$\mathbf{V}^{(m+1)} = \mathbf{V}^{(m)} + \mathbf{z} \quad (6)$$

$$\mathbf{F}'(\mathbf{V}) = \mathbf{AG}'(\mathbf{A}^T \mathbf{V})\mathbf{A}^T$$

- Calculul Jacobianului necesită evaluarea conductanțelor dinamice!
- Evaluarea conductanțelor dinamice depinde de modul în care au fost definite caracteristicile neliniare.

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## Semnificația iterațiilor Newton

Iterații Newton:

$$\mathbf{F}'(\mathbf{V}^{(m)})\mathbf{z} = -\mathbf{F}(\mathbf{V}^{(m)}) \quad (7)$$

$$\mathbf{V}^{(m+1)} = \mathbf{V}^{(m)} + \mathbf{z} \quad (8)$$

$$\mathbf{F}(\mathbf{V}) = \mathbf{AG}(\mathbf{A}^T\mathbf{V})$$

$$\mathbf{F}'(\mathbf{V}) = \mathbf{AG}'(\mathbf{A}^T\mathbf{V})\mathbf{A}^T$$

$$\mathbf{AG}'(\mathbf{A}^T\mathbf{V}^{(m)})\mathbf{A}^T\mathbf{z} = -\mathbf{AG}(\mathbf{A}^T\mathbf{V}^{(m)}) \quad (9)$$

Liniare (SRC)

$$\mathbf{AGA}^T\mathbf{V} = -\mathbf{A}\mathbf{j}$$

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## Semnificația iterațiilor Newton

Iterații Newton:

$$\mathbf{F}'(\mathbf{V}^{(m)})\mathbf{z} = -\mathbf{F}(\mathbf{V}^{(m)}) \quad (7)$$

$$\mathbf{V}^{(m+1)} = \mathbf{V}^{(m)} + \mathbf{z} \quad (8)$$

$$\mathbf{F}(\mathbf{V}) = \mathbf{AG}(\mathbf{A}^T\mathbf{V})$$

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$$\mathbf{AG}'(\mathbf{A}^T\mathbf{V}^{(m)})\mathbf{A}^T\mathbf{z} = -\mathbf{AG}(\mathbf{A}^T\mathbf{V}^{(m)}) \quad (9)$$

Liniare (SRC)

$$\mathbf{AGA}^T\mathbf{V} = -\mathbf{A}\mathbf{j}$$

Semnificația relației (9):

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## Semnificația iterațiilor Newton

Iterații Newton:

$$\mathbf{F}'(\mathbf{V}^{(m)})\mathbf{z} = -\mathbf{F}(\mathbf{V}^{(m)}) \quad (7)$$

$$\mathbf{V}^{(m+1)} = \mathbf{V}^{(m)} + \mathbf{z} \quad (8)$$

$$\mathbf{F}(\mathbf{V}) = \mathbf{AG}(\mathbf{A}^T\mathbf{V})$$

$$\mathbf{F}'(\mathbf{V}) = \mathbf{AG}'(\mathbf{A}^T\mathbf{V})\mathbf{A}^T$$

$$\mathbf{AG}'(\mathbf{A}^T\mathbf{V}^{(m)})\mathbf{A}^T\mathbf{z} = -\mathbf{AG}(\mathbf{A}^T\mathbf{V}^{(m)}) \quad (9)$$

Liniare (SRC)

$$\mathbf{AGA}^T\mathbf{V} = -\mathbf{A}\mathbf{j}$$

Semnificația relației (9):

La fiecare iterație se rezolvă un circuit liniar, potențialele lui reprezintă corecțiile în iterațiile Newton

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## Semnificația iterațiilor Newton

Iterații Newton:

$$\mathbf{F}'(\mathbf{V}^{(m)})\mathbf{z} = -\mathbf{F}(\mathbf{V}^{(m)}) \quad (7)$$

$$\mathbf{V}^{(m+1)} = \mathbf{V}^{(m)} + \mathbf{z} \quad (8)$$

$$\mathbf{F}(\mathbf{V}) = \mathbf{AG}(\mathbf{A}^T\mathbf{V})$$

$$\mathbf{F}'(\mathbf{V}) = \mathbf{AG}'(\mathbf{A}^T\mathbf{V})\mathbf{A}^T$$

$$\mathbf{AG}'(\mathbf{A}^T\mathbf{V}^{(m)})\mathbf{A}^T\mathbf{z} = -\mathbf{AG}(\mathbf{A}^T\mathbf{V}^{(m)}) \quad (9)$$

Liniare (SRC)

$$\mathbf{AGA}^T\mathbf{V} = -\mathbf{A}\mathbf{j}$$

Semnificația relației (9):

La fiecare iterație se rezolvă un circuit liniar, potențialele lui reprezintă corecțiile în iterațiile Newton

*Circuit incremental*

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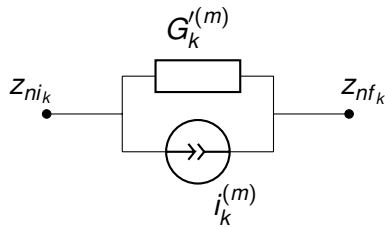
## Circuite incrementale/liniarizate

Neliniar

$$\mathbf{AG}'(\mathbf{A}^T \mathbf{V}^{(m)}) \mathbf{A}^T \mathbf{z} = -\mathbf{AG}(\mathbf{A}^T \mathbf{V}^{(m)})$$

Liniar

$$\mathbf{AGA}^T \mathbf{V} = -\mathbf{A} \mathbf{j}$$



$$z_{ni_k} = V_{ni_k}^{(m+1)} - V_{ni_k}^{(m)} \quad z_{nf_k} = V_{nf_k}^{(m+1)} - V_{nf_k}^{(m)}$$

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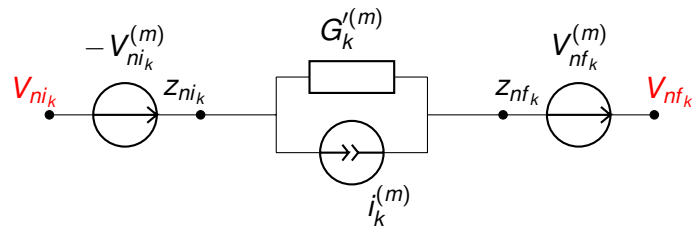
## Circuite incrementale/liniarizate

Neliniar

$$\mathbf{AG}'(\mathbf{A}^T \mathbf{V}^{(m)}) \mathbf{A}^T \mathbf{z} = -\mathbf{AG}(\mathbf{A}^T \mathbf{V}^{(m)})$$

Liniar

$$\mathbf{AGA}^T \mathbf{V} = -\mathbf{A} \mathbf{j}$$



$$z_{ni_k} = V_{ni_k}^{(m+1)} - V_{ni_k}^{(m)} \quad z_{nf_k} = V_{nf_k}^{(m+1)} - V_{nf_k}^{(m)}$$

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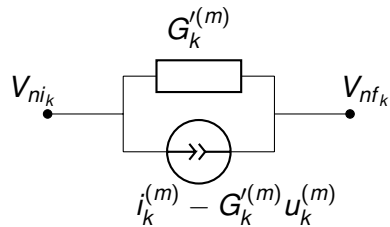
## Circuite incrementale/liniarizate

Neliniar

$$\mathbf{AG}'(\mathbf{A}^T \mathbf{V}^{(m)}) \mathbf{A}^T \mathbf{z} = -\mathbf{AG}(\mathbf{A}^T \mathbf{V}^{(m)})$$

Liniar

$$\mathbf{AGA}^T \mathbf{V} = -\mathbf{A}\mathbf{j}$$



Circuit liniarizat →

La fiecare iterație se rezolvă un circuit liniar, potențialele lui reprezintă soluțiile noi în iterațiile Newton

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## Algoritm - bazat pe asamblare de circuite

Idea (nr. 1):

Se rezolvă o succesiune de circuite rezistive liniare (liniarizate).

it = 0

inițializează soluția  $\mathbf{V}$

repetă

it = it + 1

înlocuiește elementele neliniare cu schemele lor *liniarizate*

rezolvă circuitul rezistiv liniar și calculează  $\mathbf{Vn}$

actualizează soluția  $\mathbf{V} = \mathbf{Vn}$

dacă it == itmax scrie mesaj de eroare

cât timp  $\text{norma}(\mathbf{V} - \mathbf{Vnou}) > \text{toleranța impusă}$  și it < itmax

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## Algoritm - bazat pe rezolvare de circuite

Idea (nr. 2):

Se rezolvă o succesiune de circuite rezistive liniare (incrementale).

it = 0

inițializează soluția  $\mathbf{V}$

repetă

it = it + 1

înlocuiește elementele neliniare cu schemele lor *incrementale*

rezolvă circuitul rezistiv liniar și calculează corecțiile  $\mathbf{z}$

actualizează soluția  $\mathbf{V} = \mathbf{V} + \mathbf{z}$

dacă it == itmax scrie mesaj de eroare

cât timp norma( $\mathbf{z}$ ) > toleranța impusă și it < itmax

## Algoritm - bazat pe operații cu matrice

Idea (nr. 3):

Se rezolvă o succesiune de sisteme algebrice liniare.

it = 0

asamblează matricea  $\mathbf{A}$

inițializează soluția  $\mathbf{V}$

repetă

it = it + 1

calculează conductanțele dinamice și asamblează  $\mathbf{G}'$

rezolvă sistemul liniar  $\mathbf{AG}'\mathbf{A}^T\mathbf{z} = -\mathbf{Ai}$  și calculează corecțiile  $\mathbf{z}$

actualizează soluția  $\mathbf{V} = \mathbf{V} + \mathbf{z}$

dacă it == itmax scrie mesaj de eroare

cât timp norma( $\mathbf{z}$ ) > toleranța impusă și it < itmax

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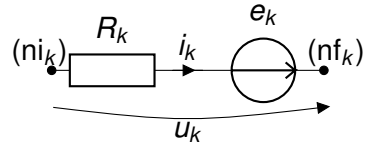
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## Cel mai simplu algoritm - pe ce ne bazăm

Primul algoritm scris pentru circuite rezistive liniare (crl) - laturi SRT



```
; declaratii date - varianta A
intreg N           ; număr de noduri
intreg L           ; număr de laturi
tablou intreg ni[L] ; noduri inițiale ale laturilor
tablou intreg nf[L] ; noduri finale ale laturilor
tablou real R[L]   ; rezistențe
tablou real e[L]   ; tensiuni electromotoare
```

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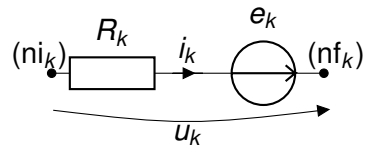
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## Cel mai simplu algoritm - pe ce ne bazăm

Primul algoritm scris pentru circuite rezistive liniare (crl) - laturi SRT



```
; declarații date - varianta B
inregistrare circuit
intreg N           ; număr de noduri
intreg L           ; număr de laturi
tablou intreg ni[L] ; noduri inițiale ale laturilor
tablou intreg nf[L] ; noduri finale ale laturilor
tablou real R[L]   ; rezistențe
tablou real e[L]   ; tensiuni electromotoare
```

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## Cel mai simplu algoritm - pe ce ne bazăm

Să pp că avem la dispoziție o procedură:

procedură nodal\_crl(circuit,v)

; rezolvă un circuit rezistiv liniar cu metoda nodală

; date de intrare: structura circuit

; ieșire: valorile potențialelor  $v$  în noduri, ultimul nod este de referință

...

retur

Obs: procedura cuprinde atât asamblarea sistemului de ecuații cât și rezolvarea lui.

## Cel mai simplu algoritm - ce e nou

- Admitem acum în plus, laturi rezistive neliniare, controlate în tensiune;

Vom presupune că există câte o procedură care poate, pentru orice latură neliniară, să întoarcă

- curentul prin latură pentru o tensiune dată ( $i_k = g_k(u_k)$ );  
Dacă curbele neliniare sunt date tabelar - aceasta presupune o **interpolare**).
- conductanța dinamică a laturii, pentru o tensiune dată ( $G'_k = g'_k(u_k)$ ).  
Dacă curbele neliniare sunt date tabelar - aceasta presupune o **derivare numerică**).

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## Cel mai simplu algoritm - etapa de preprocesare

```
functie citire_date ()  
; declarații  
...  
citește circuit.N, circuit.L  
pentru k = 1, circuit.L  
    citește circuit.nik, circuit.nfk  
    citește circuit.tipk ; tipul poate fi "R" sau "n"  
    dacă circuit.tipk = "R"  
        citește circuit.ek, circuit.Rk  
    •  
citește tol ; toleranță pentru procedura Newton  
citește itmax ; numărul maxim de iterații admis  
•  
întoarce circuit
```

## Cel mai simplu algoritm - etapa de preprocesare

```
functie citire_date ()  
; declarații  
...  
citește circuit.N, circuit.L  
pentru k = 1, circuit.L  
    citește circuit.nik, circuit.nfk  
    citește circuit.tipk ; tipul poate fi "R" sau "n"  
    dacă circuit.tipk = "R"  
        citește circuit.ek, circuit.Rk  
    •  
citește tol ; toleranță pentru procedura Newton  
citește itmax ; numărul maxim de iterații admis  
•  
întoarce circuit
```

Dar partea neliniară?

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## Cel mai simplu algoritm - etapa de preprocesare

Variante - pentru partea neliniară:

```
funcție g(u)        funcție g(u)
Is = 1e-12          nd = 3 ; numărul de puncte de discontinuitate
Vt = 0.0278        uval = .....
                    ival = ....
                    m = cauta(uval, ival, u)
întoarce Is*(exp(u/Vt)-1) întoarce ival(m) + (ival(m+1) - ival(m))/(uval(m+1)-uval(m))*(u - uval(m))
```

```
funcție gder(u)     funcție gder(u)
Is = 1e-12          nd = 3 ; numărul de puncte de discontinuitate
Vt = 0.0278        uval = .....
                    ival = ....
                    m = cauta(uval, ival, u)
întoarce Is*exp(u/Vt)/Vt întoarce (ival(m+1) - ival(m))/(uval(m+1)-uval(m))
```

Is, Vt, nd, uval, ival - pot fi citite în etapa de preprocesare (și pot fi diferite pentru diferitele elemente neliniare).

## Algoritm - v2

```
procedură solve_crn1_v2(circuit, tol, itmax, V)
circuit - structură - parametru de intrare
tol, itmax - parametri de intrare, specifici procedurii Newton
V - vector - parametru de ieșire
....
inițializare
V = 0 ; vector de dimensiune N
err = 1
itk = 0
cât timp err > tol și itk < itmax
  kit = kit + 1
  pentru k = 1:L
    dacă circuit.tip(k) == "n"
      tens = V(circuit.ni(k)) - V(circuit.nf(k))
      cond_din = gder(tens)
      crt = g(tens)
      circuit.R(k) = 1/cond_din
      circuit.e(k) = circuit.R(k)*crt - tens
    •
    •
    •
  nodal_cr1(circuit, Vn)
  err = norma(Vn - V)
  V = Vn
  •
return
```

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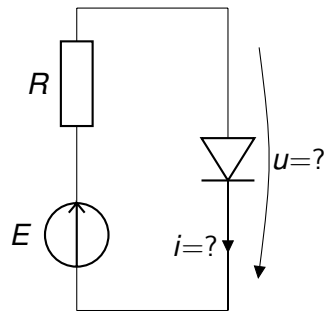
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## Algoritm - v1

```
procedură solve_crn1_v1(circuit,tol,itmax,V)
circuit - structură - parametru de intrare
tol, itmax - parametri de intrare, specifici procedurii Newton
V - vector - parametru de ieșire
....
inițializare
V = 0 ; vector de dimensiune N
err = 1
itk = 0
cât timp err > tol și itk < itmax
    kit = kit + 1
    pentru k = 1:L
        dacă circuit.tip(k) == "n"
            tens = V(circuit.ni(k)) - V(circuit.nf(k))
            cond_din = gder(tens)
            crt = g(tens)
            circuit.R(k) = 1/cond_din
            circuit.e(k) = circuit.R(k)*crt
        •
        •
        nodal_cr(circuit,z)
        err = norma(z)
        V = V + z
    •
retur
```

## Exemplul 1 - rezultate



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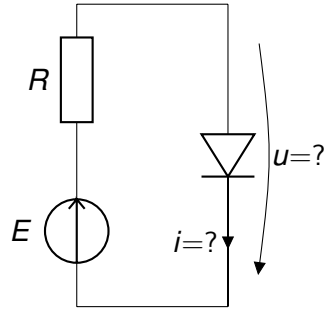
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## Exemplul 1 - rezultate



$$i = g(u)$$
$$i = \frac{E - u}{R}$$

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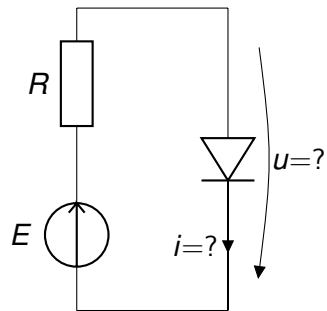
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## Exemplul 1 - rezultate



$$i = g(u)$$
$$i = \frac{E - u}{R}$$
$$E = 1.25V, R = 1.25m\Omega$$

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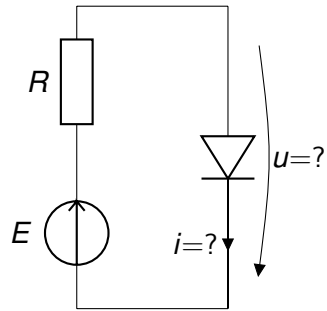
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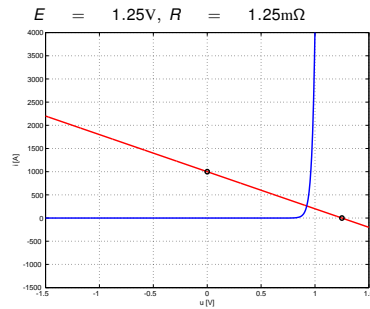
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## Exemplul 1 - rezultate

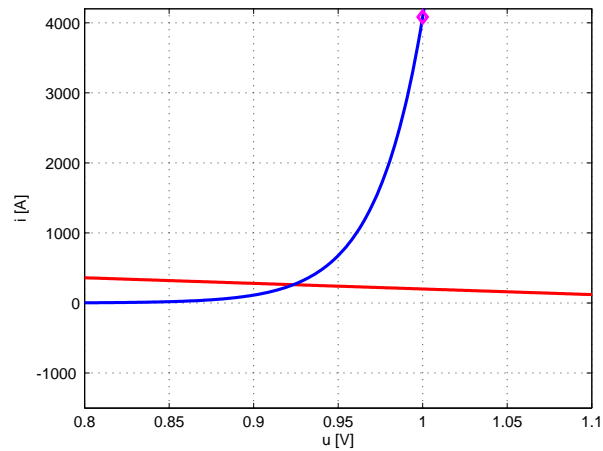


$$i = g(u)$$

$$i = \frac{E - u}{R}$$



## Exemplul 1 - rezultate



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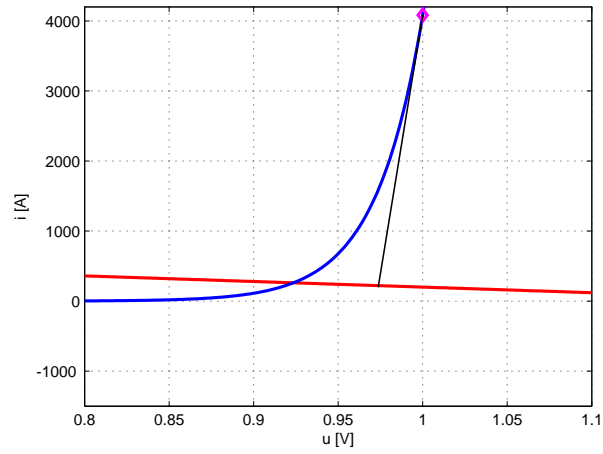
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## Exemplul 1 - rezultate



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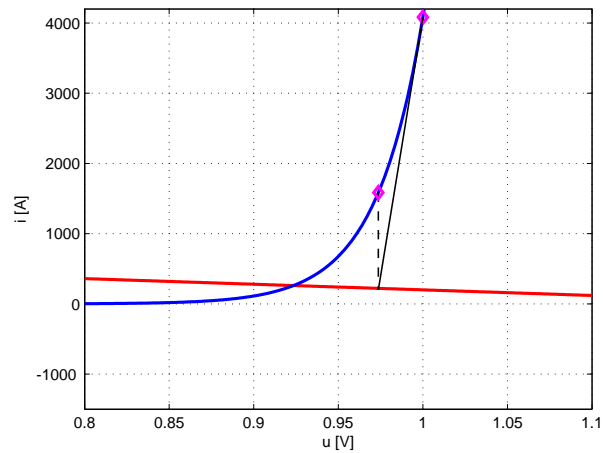
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## Exemplul 1 - rezultate



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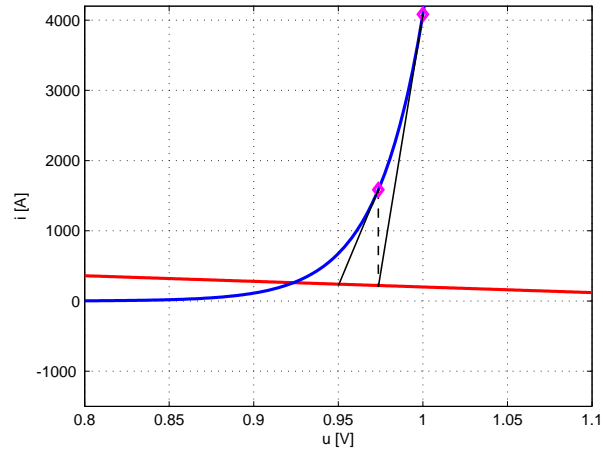
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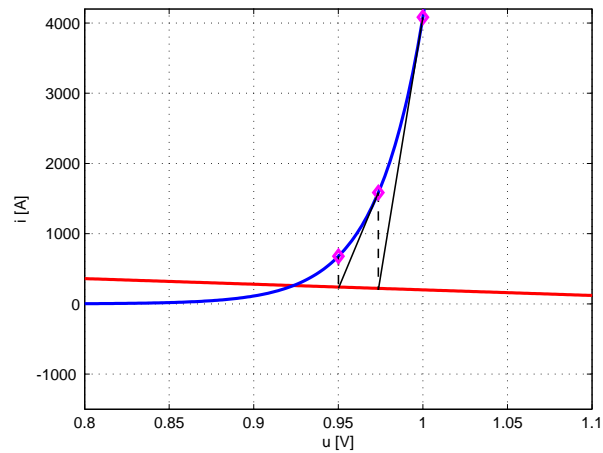
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## Exemplul 1 - rezultate



## Exemplul 1 - rezultate



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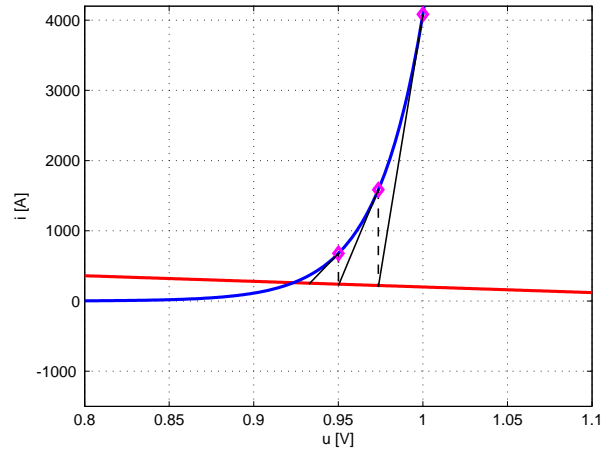
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## Exemplul 1 - rezultate



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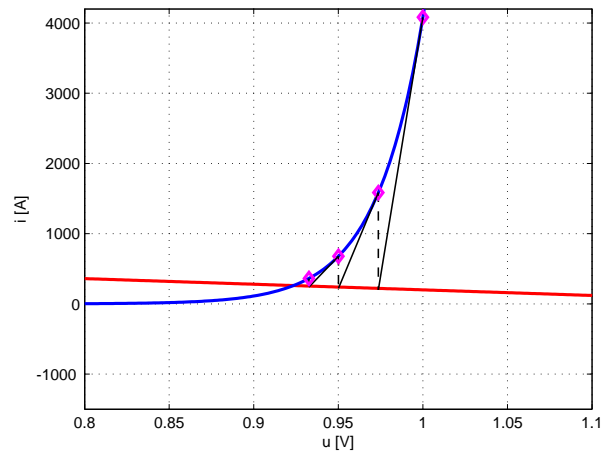
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## Exemplul 1 - rezultate



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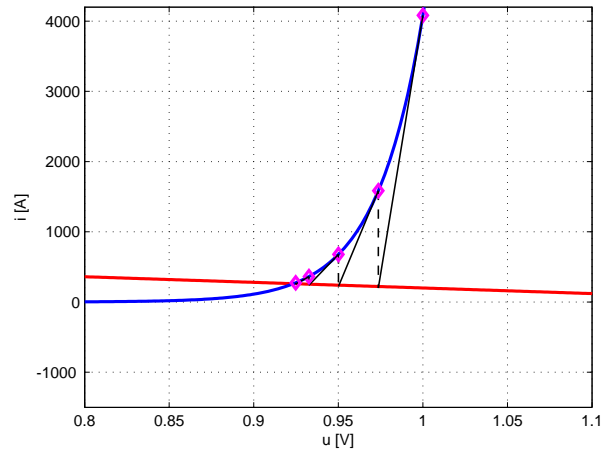
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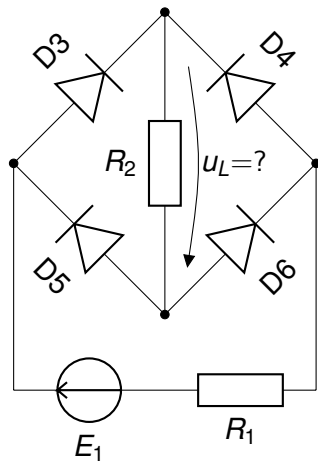
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## Exemplul 1 - rezultate



## Exemplul 4 - rezultate



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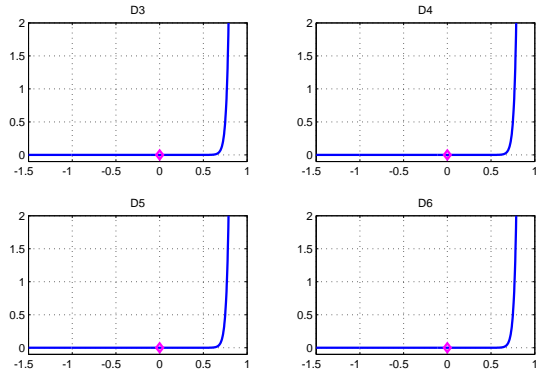
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## Exemplul 4 - rezultate

$E_1 = 2V$ ,  $R_1 = 1\Omega$ ,  $R_2 = 2\Omega$ , 13 iterații pentru  $\text{tol} = 0.01$

Numai inițializarea și ultimele patru sunt ilustrate.



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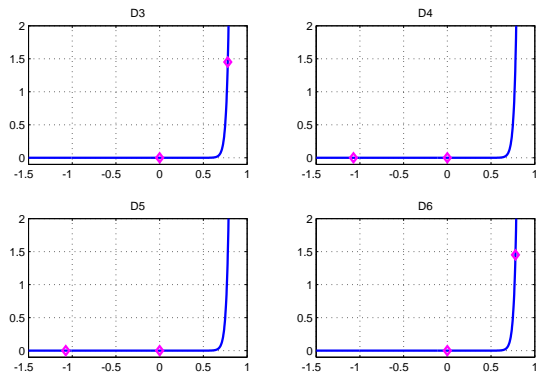
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## Exemplul 4 - rezultate

$E_1 = 2V$ ,  $R_1 = 1\Omega$ ,  $R_2 = 2\Omega$ , 13 iterații pentru  $\text{tol} = 0.01$

Numai inițializarea și ultimele patru sunt ilustrate.



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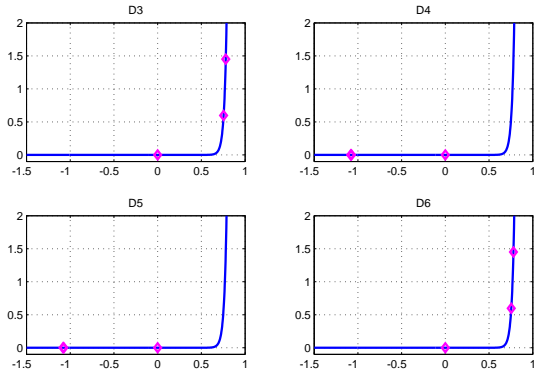
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## Exemplul 4 - rezultate

$E_1 = 2V$ ,  $R_1 = 1\Omega$ ,  $R_2 = 2\Omega$ , 13 iterații pentru  $\text{tol} = 0.01$

Numai inițializarea și ultimele patru sunt ilustrate.

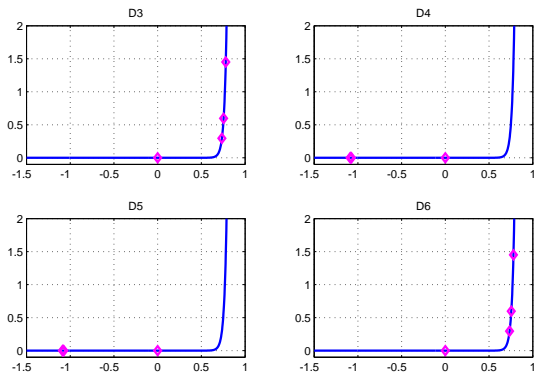


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## Exemplul 4 - rezultate

$E_1 = 2V$ ,  $R_1 = 1\Omega$ ,  $R_2 = 2\Omega$ , 13 iterații pentru  $\text{tol} = 0.01$

Numai inițializarea și ultimele patru sunt ilustrate.



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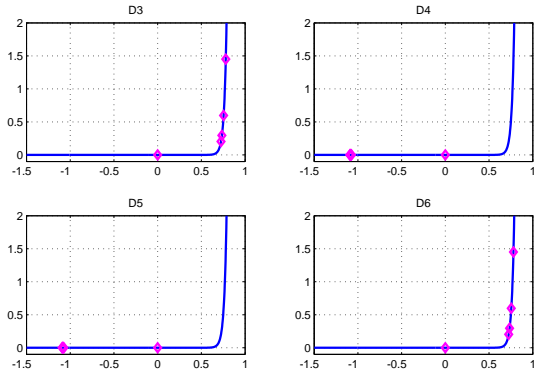
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## Exemplul 4 - rezultate

$E_1 = 2V$ ,  $R_1 = 1\Omega$ ,  $R_2 = 2\Omega$ , 13 iterații pentru  $\text{tol} = 0.01$

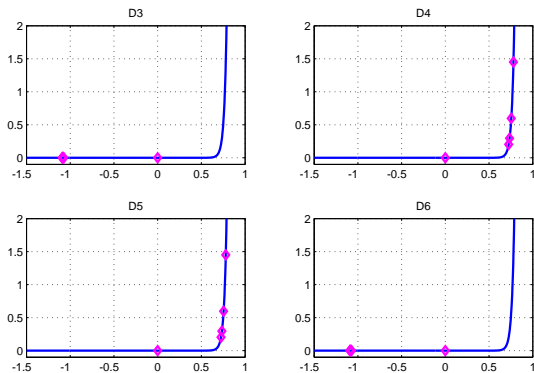
Numai inițializarea și ultimele patru sunt ilustrate.



## Exemplul 4 - rezultate

$E_1 = -2V$ ,  $R_1 = 1\Omega$ ,  $R_2 = 2\Omega$ , 13 iterații pentru  $\text{tol} = 0.01$

Numai inițializarea și ultimele patru sunt ilustrate.



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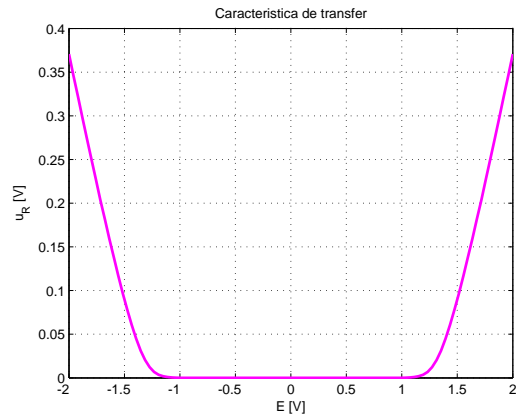
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## Exemplul 4 - rezultate

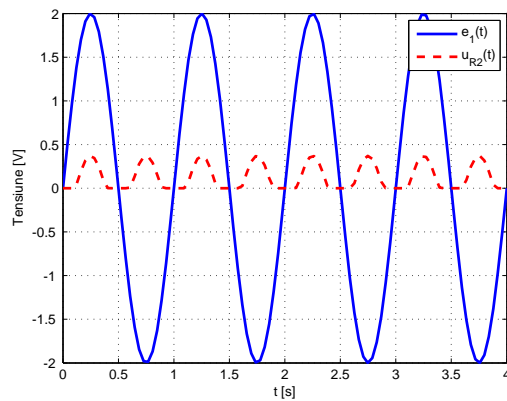
$E_1 \in [-2, 2]V$ ,  $R_1 = 1\Omega$ ,  $R_2 = 2\Omega$ ,  $u_{R2} = ?$



## Exemplul 4 - rezultate

Sursa variabilă în timp? *Timpul are un caracter convențional. (Sistemul este algebric!)*

$e_1(t) = 2 \sin(2\pi t)V$ ,  $R_1 = 1\Omega$ ,  $R_2 = 2\Omega$ ,  $u_{R2}(t) = ?$



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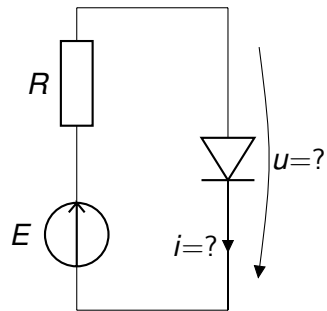
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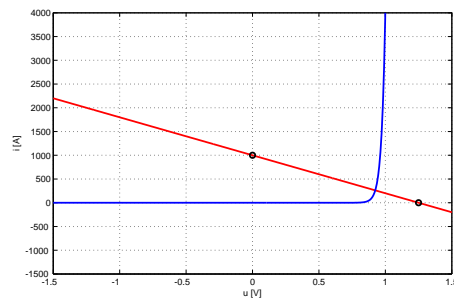
## Concluzii

- Analiza circuitelor rezistive neliniare se reduce la o succesiune de rezolvări de sisteme algebrice liniare (care pot fi privite ca rezolvări de circuite rezistive liniare - incrementale sau liniarizate).
- Convergența procedurii depinde de inițializare.
- Numărul de iterații depinde de inițializare și de eroarea impusă soluției.

## Cazul caracteristicilor Ipp



Aproximația Ipp a caracteristicii diodei semiconductoare.



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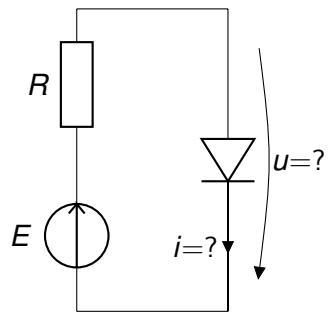
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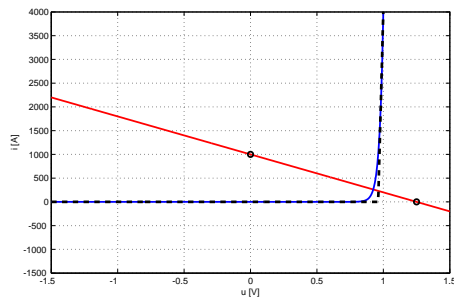
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## Cazul caracteristicilor Ipp

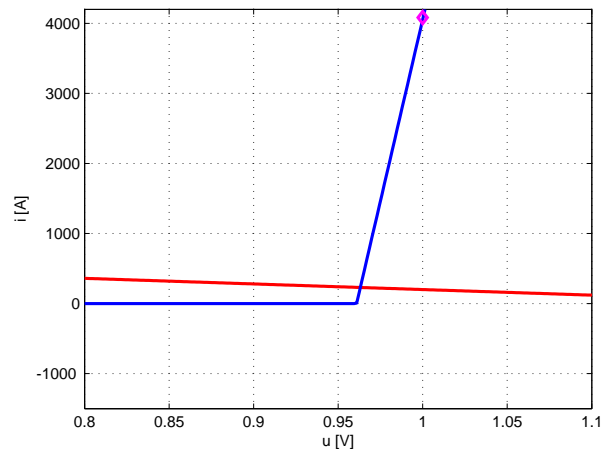


Aproximația Ipp a caracteristicii diodei semiconductoare.



## Cazul caracteristicilor Ipp

Iterații Newton - inițializarea.



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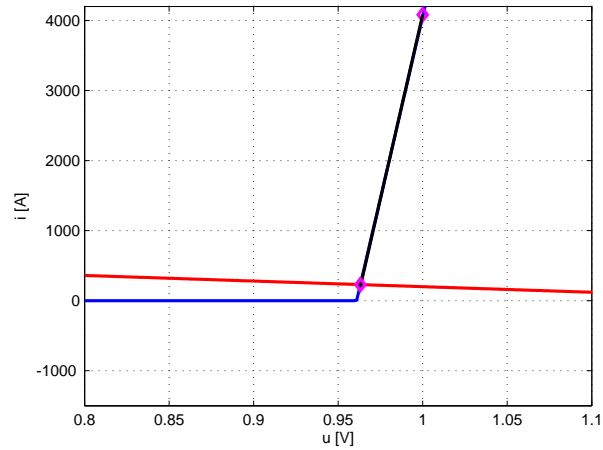
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## Cazul caracteristicilor Ipp

Iterații Newton - iterația 1.



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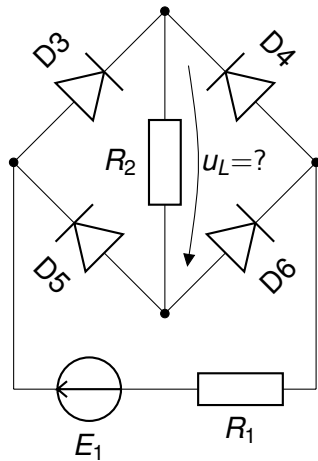
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## Cazul caracteristicilor Ipp



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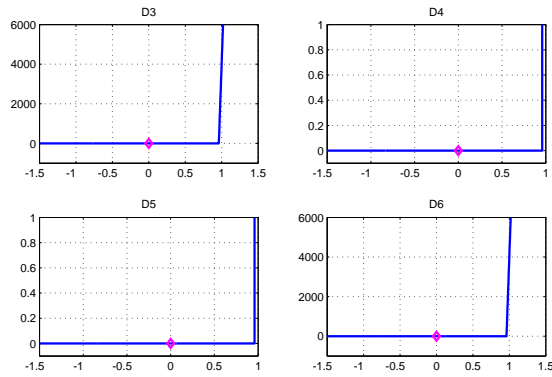
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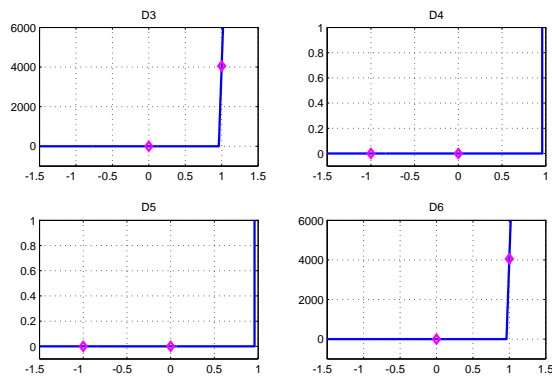
# Cazul caracteristicilor Ipp

Iterații Newton - inițializarea.



# Cazul caracteristicilor Ipp

Iterații Newton - iterația 1.



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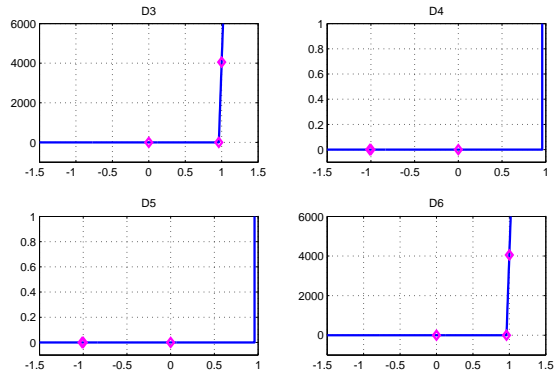
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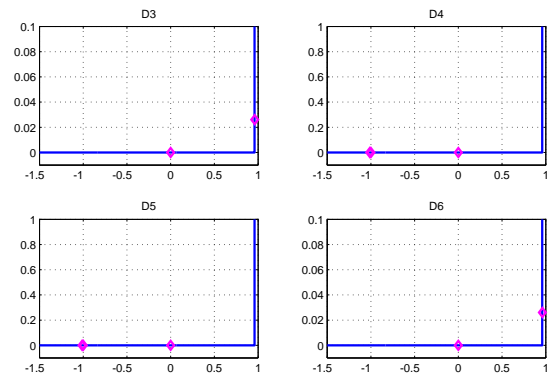
## Cazul caracteristicilor Ipp

Iterații Newton - iterația 2.



## Cazul caracteristicilor Ipp

Iterații Newton - iterația 2 - zoom in.



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## Cazul caracteristicilor Ipp

- Eroarea impusă nu influențează prea mult numărul de iterații deoarece după determinarea corectă a segmentului în care se află PSF, eroarea impusă este satisfăcută la următoarea iterație.
- Dacă inițializarea corespunde combinației corecte de segmente, atunci se va face exact o singură iterație.
- Numărul maxim de iterații este egal cu numărul maxim de combinații de segmente.
- Există o variantă a metodei (cunoscută sub numele de metoda Katzenelson) în care la fiecare iterație se modifică un singur segment, cel corespunzător variației maxime. Avantaj - convergența garantată.

## Lectură

Obligatoriu:

Ioan98 D. Ioan et al., *Metode numerice in ingineria electrica*, Ed. Matrix Rom, Bucuresti, 1998. (Capitolul 17)

Cartea se găsește la biblioteca UPB, puteți verifica accesând catalogul <http://www.library.pub.ro/>.

Facultativ:

Chua75 Leon Chua, Pen-Min Lin, *Computer-Aided Analysis of Electronic Circuits*, Prentice-Hall, 1975. (Capitolele 5 și 7)

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